Modeling and quantification of aging systems for maintenance optimization

William Lair, PhD student, SNCF, Université Paris Est Marne la Vallée

Rachid Ziani, Project Manager, SNCF

Sophie Mercier, Professor, Université de Pau

Michel Roussignol, Professor, Université Paris Est Marne la Vallée

Key Words: Markov, PDMP, dynamic reliability, maintenance optimization, finite volumes

SUMMARY

This article deals with the maintenance optimization of a train air conditioning system. Indeed, SNCF (French Railway Company) has in charge the maintenance of its material.

In order to model this system, we use dynamic reliability method, the Piecewise Deterministic Markov Processes (PDMP). A deterministic method is used to calculate the reliability quantities : the finite volumes algorithm.

The results found in this study are confidential, so we present results computed with fictive costs and laws. Thanks to this method, we have found a strategy which reduces the maintenance cost of 7% and the system failures number of 22%. Moreover, we observe that in this case, the finite volumes algorithm is faster than the Monte Carlo simulations.

1 INTRODUCTION

For a railway infrastructure manager as the SNCF, materiel maintenance constitutes a major task : a materiel failure is expensive and causes customers dissatisfaction. The SNCF has hence initiated researches in order to model aging systems, in view of its facilities maintenance. This article deals with an air conditioning system case.

The air conditioning system is a parallel system consisting of seventeen aging components. Seven of them compel the system to crash if they fail. The rest is located on two identical circuits that operate at the same time. The failure of one circuit does not coerce the system to break. However if the both of them collapse, the system also does. The components lifetime distributions are Weibull's : this signifies that the components age.

Because of the components aging, usual Markov processes like Markov jump processes can't be used. Consequently, in order to model the air conditioning system, we work with Markov processes named Piecewise Deterministic Markov Processes (PDMP), see [1] and [2]. The reliability calculations are often established by Monte Carlo simulations, see [3] and [4] ; however this method takes usually too much time to optimize maintenance. Thus, we use a finite volumes algorithm which solves the Chapman-Kolmogorov equations. Indeed, this is a deterministic method which may be faster in this case.

In the first part, the methods we use to optimize the maintenance are presented : the PDMP, the finite volumes algorithm, and the simulated annealing algorithm. Then the air conditioning system, its modelling by a PDMP, and the results of the optimization are described. In a third part, the computation time with the Monte Carlo simulations and with the finite volumes algorithm are compared. In the end, some conclusions are given.

2 DESCRIPTION OF THE METHODOLOGY

2.1 *Piecewise Deterministic Markov Process*

The PDMP is a process exploited in dynamic reliability which has been introduced by Davis in 1984 [1],[2]. It is a hybrid process $(I_t,X_t)_{t>0}$. The first component I_t is discrete, with values in a finite state space E. Typically, it indicates the state - up or down - for each component of the system at time t. The second component X_t, with values in R^d, stands for environmental conditions, such as temperature, pressure, and in our case, the system components ages. This means that PDMP can model a system with aging components. The two components interact with each other : the process jumps at many countable isolated random times. By a jump from $(I_{t^-}, X_{t^-})=(i,x)$ to $(I_t,X_t)=(j,y)$ (with $(i,x),(j,y)\in E\times \mathrm{R}^d$), the transition rate between the discrete states i and j depends on the environmental condition just before the jump x, and is a function $x\rightarrow a(i,j,x)$. Similarly, the environmental condition just after the jump, X_t, is distributed according to some distribution $\mu_{(i,j,x)}(dy)$, which depends on both components just before the jump (i,x) and on the after jump discrete state j. Between jumps, the discrete component I_t is constant and the evolution of environmental condition X_t is deterministic, solution of a set of differential equations which depends on the fixed state : given $I_t=i$ for all $t\in [a,b]$, we have

$$\frac{d}{dt}X_t = v(i,X_t) \tag{1}$$

In addition, when X_t reaches a border called Γ , the process jumps according the distribution $q_{(i,j,x)}(dy)$ with $x\in \Gamma$, which depends on the component before the jump, i, and after the jump, j. In this article, the border represents the time we execute preventive maintenance.

The reliability quantities we search, are calculated using PDMP distributions at time t. T*o* calculate these quantities, Monte-Carlo simulations can be used. However this method generates large computation time. In this paper, we use a direct calculation method which needs the PDMP marginal distributions. These ones at time t, noted $\pi_t(i,x)$, constitute the unique solution of a set of explicit integro-differential equations named Chapman-Kolmogorov equations [5]. The issue is that the C-K equations are often impossible to directly solve. This article proposes another approach which delivers an approximation of the C-K equations solutions with a finite volumes algorithm [6], [7] and [8].

2.2 *The finite volumes algorithm*

This algorithm calculates an approximation of PDMP marginal distributions. The principle is based on discretization of the time and of the environmental variable state space. The distribution evolution of the process is followed and the probability that the process is in a mesh, is computed time step by time step. This brings us to solve a linear system.

Its computation time depends on one parameter : the discretization step of the environmental variable state space, noted h. The smaller the step is, the longer the computation time is and the more precise the results are. The calculations are executed with different values of h. The time discretization step is the maximum value that makes the algorithm converge.

We use this algorithm to test different maintenance strategies. The first one is a preventive maintenance that occurs at time T in which too old components are changed. A component is considered as too old if during the maintenance, it is older than a limit age defined by the maintenance schedule. In order to optimize this strategy, we search the preventive maintenance instant and the components limit age. The second one is the opportunistic maintenance strategy. When the system crashes, the broken and too old components are replaced by new ones. However there are too many ways to apply these two strategies so all of them can't be tested ; that is why we use an optimization algorithm : the simulated annealing algorithm.

2.3 The simulated annealing algorithm

The simulated annealing algorithm [9] is a stochastic algorithm that finds a global minimum of a function. It's used when the optimum can not be directly found. This is the case when the function has too many variables.

Wikipedia website gives a fine explanation of the algorithm : "The name and inspiration come from annealing in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects. The heat causes the atoms to become unstuck from their initial positions (a local minimum of the internal energy) and to randomly wander through states of higher energy. The slow cooling brings them more chances to find configurations with lower internal energy than the initial one.

By analogy with this physical process, each step of the SA algorithm replaces the current solution by a random "nearby" solution, chosen with a probability that depends on the difference between the corresponding function values and on a global parameter Te - called the temperature - that is gradually decreased during the process. The dependency is such that the current solution almost randomly changes as Te is large, but increasingly "downhill" as Te goes to zero. The allowance for "uphill" moves saves the method from becoming stuck at local minima - which is the bane of greedier methods."

3 THE AIR CONDITIONING SYSTEM

3.1 Description of the system

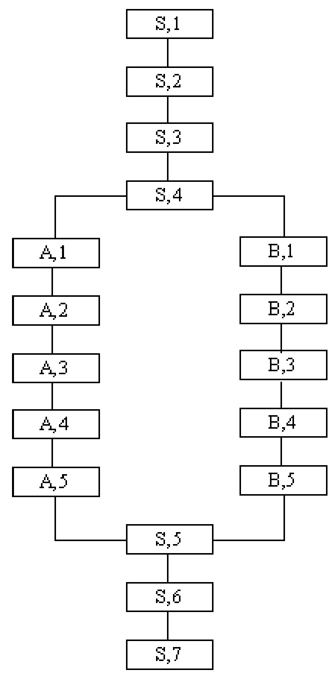

Figure 1 – Diagram of the air conditioning system

	Shape	Scale	Cost
S,1	1.5	30	300
S,2	2	20	400
S,3	1.5	80	1000
S,4	2.5	50	800
S,5	1.2	60	250
S,6	2	20	400
S,7	3	40	300
A-B,1	2.5	35	200
A-B,2	1.3	25	1000
A-B,3	2	50	400
A-B,4	1.5	45	300
A-B,5	1.8	20	200

Table 1 - Weibull distributions coefficient and cost of the components

The figure 1 describes the air conditioning system. It has seventeen aging components. A part of it is in active redundancy and the rest is in series. The first part has two circuits called A and B. There are five components on circuit A and five on circuit B. Circuit A and circuit B are identical. The two branches work together. When a component on one of these branches fails, the components on the same branch stop aging. The system crashes if one component in series fails or if one component on circuit A and one component on circuit B fail. When the air conditioning runs out, it is instantly repaired. The restoration consists on replacing the broken with new ones.

The components lifetime distributions are Weibull's. The Weibull distributions coefficients and the components cost are exposed in table 1. The data are confidential so the table 1 does deliver fictitious figures.

3.2 *Maintenance strategies*

We try to establish two types of maintenance. The first one is making a single preventive maintenance, and during this review, the components that are older than specified age are changed. In order to optimize this strategy, we must look on thirteen parameters. One is the system age we need to practice preventive maintenance, and the rest corresponds to the components limit ages. If a component is older than its limit age at the review time, it is replaced by a new one. The second one is the opportunistic maintenance strategy. It is about taking advantage of a failure by simultaneously changing too old components, in addition to the broken components. In order to optimize this strategy, we have to look on twelve parameters, corresponding to the limit ages of the components. We optimize the maintenance system mean cost. The table 2 provides the cost of preventive and corrective maintenances.

Preventive maintenance cost	Corrective maintenance cost
500	2000

Table 2 - Cost of maintenance

3.3 *Modelling by a PDMP*

Let E be the space state of the discrete process of the PDMP, E is $\{0,1\}^{17}$ (0 for down, 1 for up). Thus I_t models the states of all components at time t. Actually the process occurs in a slight part of E which is:

- 1: "all the components of the system work"
- $1_{K,i}$: "the system works, but the component (K,i) is down" with $K\in \{A,B\}$.
- $0_{S,i}$: "The system has failed, the component (S,i) is down"
- $0_{K,i;L,j}$: "the system has failed, the components (K,i) and (L,j) are down" with $K\in \{A,B\}$ and $L\in \{A,B,S\}$

X_t describes the age of all components at time t. There are seventeen components, so the continuous process space state is R^{17}.

In our case, the equation (1) is simple since the environmental variables are components ages. Be $x_{K,i}$ the component (K,i) age. When component (K,i), $K\in \{A,B,S\}$, is aging $g_1(x_{K,i},t)=x_{K,i}+t$ is the solution. In some cases, when component (K,i) doesn't work, it's not aging so $g_2(x_{K,i},t)=x_{K,i}$ is the solution. For a component from part S, if it works, g_1 is the solution of (1), else it is g_2. For a component in part A or B, g_1 is the solution of (1) if this component and all the components on the same circuit work, else it is g_2.

Let's write the jump rates for the air conditioning system. In what follows, $K\in \{A,B\}$ and $L\in \{A,B,S\}$ and $K\neq L$. The process jumps when a component fails or when the system is repaired. Let's note $a_{K,i}(x_{K,i})$ the failure rate of the component (K,i) with $K\in \{A,B,S\}$. The failure rates are not constant because the components lifetime distributions are Weibull's. The transition rates between states when a component fails are:

$$a(1,0_{S,i},x)=a_{S,i}(x_{S,i})$$
$$a(1,1_{K,i},x)=a_{K,i}(x_{K,i})$$
$$a(1_{K,i},0_{K,i;L,i},x)=a_{L,j}(x_{L,j})$$

The figure 2 describes the transitions between the states of the process.

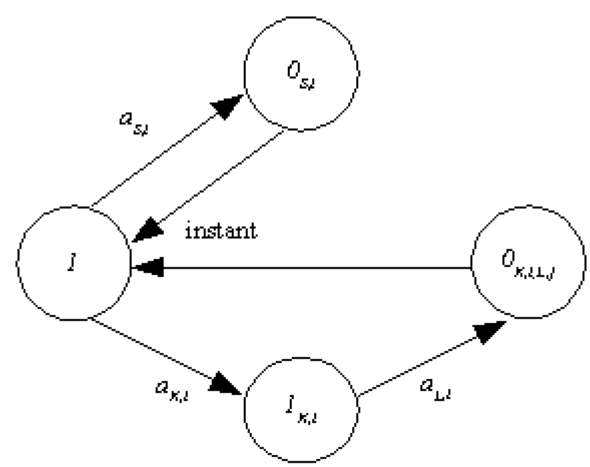

Figure 2 - Modelling of the air conditioning system

When a component fails, the components age doesn't change but when the system is repaired, the age of the components which have been changed by new ones becomes equal to 0. So the process continuous part distributions after a jump are:

$$\mu_{(1,0_{S,i},x)}(dy)=\mu_{(1,1_{K,i},x)}(dy)=\mu_{(1_{K,i},0_{K,i;L,j},x)}(dy)=\delta_x(dy)$$
$$\mu_{(0_{S,i},1,x)}(dy)=\delta_0(dy_{S,i})\prod_{(M,j)\neq(S,i)}\delta_{x_{M,j}}(dy_{M,j})$$
$$\mu_{(0_{K,i;L,j},1,x)}(dy)=\delta_0(dy_{K,i})\delta_0(dy_{L,j})\prod_{\substack{(M,k)\neq(K,i)\\(M,k)\neq(L,j)}}\delta_{x_{M,k}}(dy_{M,k})$$

If we want to model a preventive maintenance at time T, we must create a new physical variable that remembers the time since entry into service. So T is the boundary of that new variable. After this variable reaches T, it is no longer needed and the ages of the components are reset to zero if they are above their limit age. Let $u(K,i)$ be the limit age of the component (K,i), the process continuous part distributions after the time T is reached are:

$$q(1,(x,T),1) = \prod_{M,i} \left(\delta_0(dy_{M,i}) 1_{x_{M,i} \geq u_{M,i}} + \delta_{x_{M,i}}(dy_{M,i}) 1_{x_{M,i} < u_{M,i}}\right)$$

$$q(1_{K,i},(x,T),1) = \delta_0(dy_{K,i}) \prod_{(M,j) \neq (L,i)} \begin{pmatrix} \delta_0(dy_{M,j}) 1_{x_{M,j} \geq u_{M,j}} + \\ \delta_{x_{M,j}}(dy_{M,j}) 1_{x_{M,j} < u_{M,j}} \end{pmatrix}$$

To model the opportunistic maintenance, we have to change the distributions of the process after a jump. Again, let $u(K,i)$ be the limit age of the component (K,i).

$$\mu(0_{S,i},1,x) = \delta_0(dy_{S,i}) \prod_{(K,j) \neq (S,i)} \left(\delta_0(dy_{K,j}) 1_{x_{K,j} \geq u_{K,j}} + \delta_{x_{K,j}}(dy_{K,j}) 1_{x_{K,j} < u_{K,j}}\right)$$

$$\mu(0_{K,i;L,j},1,x) = \delta_0(dy_{K,i}) \delta_0(dy_{L,j}) \prod_{\substack{(M,k) \neq (K,i) \\ (M,k) \neq (L,j)}} \begin{pmatrix} \delta_0(dy_{M,k}) 1_{x_{M,k} \geq u_{M,k}} + \\ \delta_{x_{M,k}}(dy_{M,k}) 1_{x_{M,k} < u_{M,k}} \end{pmatrix}$$

3.4 *Cost function*

A maintenance strategy is optimized on the system maintenance cost on a period of thirty years. Are considered : the system failure cost C_d, the component (M,k) replacement cost $C_{(M,k)}$ and the preventive maintenance cost C_{PM}. Be N_d, the system failures number and $N_{(M,k)}$, the replacement number of component (M,k).

The cost function is given by the following:

$$C = C_d N_d + \sum_{(M,k)} C_{(M,k)} N_{(M,k)} + C_{PM} \quad (3)$$

The simulated annealing algorithm searches the strategy which minimizes the expected value of C.

3.5 *Results of the mean cost optimization*

For calculations, the discretization step of the environmental variable space state, noted h, is one month (*1/12*). The test computation time of one strategy is 25 seconds.

The table 3 shows the component limit ages of the system. During preventive or opportunistic maintenance, if a component is older than its limit age, it is replaced by a new one. For example, with the preventive maintenance strategy, when the system reaches 16 years old, if the component (*S,1*) is older than 6 years, it is replaced by a new one.

To find those strategies, the simulated annealing algorithm had to do between 200 and 300 tests. To accelerate the calculations, we begin with a discretization step of the environmental variable space state equals to four months (*1/3*). When the algorithm approaches the solution, the step is switched to one month.

With these strategies, we reduce the maintenance cost of 7% with respect to cost without preventive and opportunist maintenance. The mean number of failures obtained with the preventive maintenance is lower than the opportunistic maintenance (see table 4) : the preventive maintenance is the most efficient.

	Limit ages	
	Preventive maintenance at 16 years	Opportunistic maintenance
S,1	6	16
S,2	4	8
S,3	Do not replace	Do not replace
S,4	Do not replace	Do not replace
S,5	Do not replace	Do not replace
S,6	5	8
S,7	11	16
A-B,1	14	16
A-B,2	Do not replace	Do not replace
A-B,3	Do not replace	Do not replace
A-B,4	Do not replace	Do not replace
A-B,5	6	13

Table 3 – Optimal strategies to minimize the mean cost

	Mean cost (€)	Mean number of failures
Without preventive or opportunistic maintenance	17293	6.4
Preventive maintenance	16029 (-7.3%)	4.97 (-22.3%)
Opportunistic maintenance	16064 (-7.1%)	5.1 (-20.3%)

Table 4 - Results of the mean cost optimization

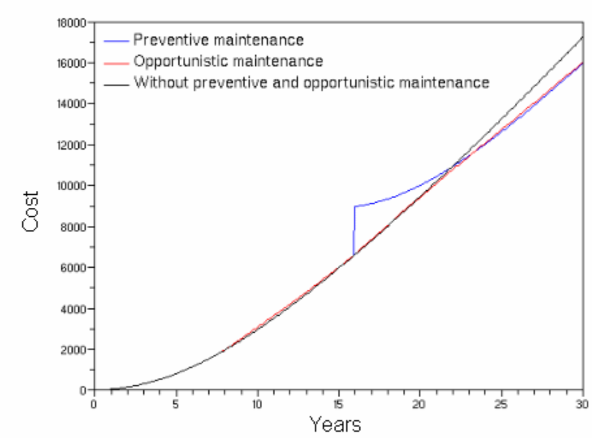

Figure 3 - Cumulative cost of the air conditioning system during 30 years

The figure 3 represents the system cumulative cost during thirty years. Preventive maintenance at sixteen years leads to a huge investment. Opportunistic maintenance appears not to cause such investments ; but actually it does : the review moment is stochastic and not deterministic.

The figure 4 is the system failure rate with and without preventive maintenance. The opportunistic one does not change this because it doesn't influence the first failure.

The figure 5 is the system failures mean number year by year.

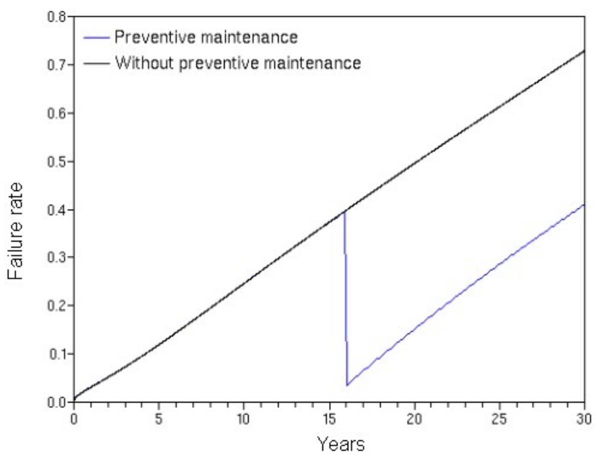

Figure 4 - Failure rate of the air conditioning system with and without preventive maintenance

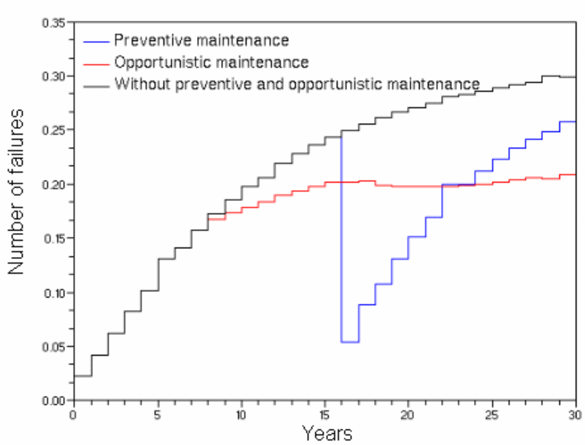

Figure 5 – Failures mean number of the air conditioning system year by year

4 COMPARISON WITH MONTE CARLO SIMULATIONS

To compare the finites volume algorithm and the Monte Carlo simulations, we make the mean cost and failures mean number calculations of the air conditioning system without preventive and opportunistic maintenance. We simulate 10^6 histories for the Monte Carlo method. To test the finite volumes algorithm, two different discretization steps of the environmental variable space state (h) are used : one month (*1/12*) and four months (*1/3*). The table 7 shows the results found with these two methods. They are similar but the finite volumes algorithm is the fastest one. The calculations with the finite volumes algorithm and a discretization step of *1/3* are executed in 3 seconds ; but the results are less accurate.

The histories number can be decreased to accelerate the calculations with the Monte Carle simulations. Nevertheless the confidence interval may be too large. Moreover if the calculations are made twice, the same results may not be obtained. These tie up the optimization.

	Mean cost (€)	Mean number of failures	CPU time
Finite volume (h=*1/12*)	17293	6.4	25
Finite volume (h=*1/3*)	17492	6.48	3
Monte Carlo	17160	6.34	1000

Table 5 – Results with the finite volume algorithm (with two different discretization steps h) and the Monte Carlo simulations

CONCLUSIONS

To conclude, this method quickly gives us reliability quantities that allow us to find an optimal maintenance. The methodology can be used for many systems. However the limitations are the number of aging components and the system complexity. Indeed, the computational time increases with those two parameters. In the future, we will try to apply this method with more complex systems and keep the computational time low. Now, we are searching importance and sensibility indicators so we can tell which component is essential.

REFERENCES

1. M.H.A. Davis, "Piecewise deterministic markov processes: a general class of non-diffusion stochastic models", *Journal Royal Statistical Soc. (B)*, vol 46, 1984, pp 353-388.
2. M.H.A. Davis, "Markov models and optimization", Chapman & Hall, 1993.
3. H. Zhang, F. Dufour, Y. Dutuit, K. Gonzalez, "Piecewise deterministic markov processes and dynamic reliability", *Journal of Risk and Reliability*, vol 222, 2008, pp 545-551.
4. P.E. Labeau, "A Monte Carlo estimation of the marginal distributions in a problem of probabilistic dynamics", *Reliability Engineering and System Safety*, vol 52, 1996, pp 65-75.

5. C. Cocozza-Thivent, R. Eymard, S. Mercier, M. Roussignol, “Characterization of the marginal distributions of markov processes used in dynamic reliability”, *Journal of Applied Mathematics and Stochastic Analysis*, vol 2006, pp 1-18.
6. C. Cocozza-Thivent, R. Eymard, S. Mercier, “A finite volume scheme of dynamic reliability models”, *IMA Journal of Numerical Analysis,* vol 26(3), 2006, pp 446-471.
7. R. Eymard, S. Mercier, A. Prignet, “An implicit volume scheme for a scalar hyperbolic problem with measure data to piecewise deterministic markov processes”, *journal of Computational and Applied Mathematics*, vol 222, 2008, pp 293-232.
8. R. Eymard, T. Gallouet, R. Herbin, “Finite Volume Methods”, *Handbook of Numerical Analysis*, vol VII, 2000, pp 713-1020.
9. M. Duflo, “Algorithmes stochastiques”, Springer, 1999.

BIOGRAPHIES

William Lair, PhD student,
Innovation & Research Department of SNCF
45 rue de Londres
75379 Paris France

e-mail : william.lair@sncf.fr

William Lair is a PhD student since January 2009. He works at the SNCF (the French national railway) in the Innovation and Research Department. The laboratory Analysis and Applied Mathematics at the Paris Est Marne la Vallée University oversees its work. His job is to optimize strategies for maintenance of facilities for SNCF. For that matter, he uses Piecewise Deterministic Markov Processes (PDMP) to model systems and do the calculations with a finite volume algorithm.

Sophie Mercier, Professor,
Université de Pau et des Pays de l'Adour
Av. de l'université - BP 576 - 64012 Pau

e-mail : sophie.mercier@univ-pau.fr

Sophie Mercier is a professor and serves on the university of Pau. Previously, she was at the Paris Est Marne la Vallée University where she was assistant professor. Most of her research is in the field of mathematics for reliability. In particular she works on dynamic reliability models described in this article. She is the thesis director of William Lair. She is one of the authors who published the algorithm on which William Lair works.

Michel Roussignol, Professor,
Université de Paris Est Marne la Vallée,
Laboratoire d'Analyse et de Mathématiques Appliquées (UMR CNRS 8050),
5, bd Descartes, Champs-sur-Marne,
77454 Marne-La-Vallée cedex 2, FRANCE

e-mail : michel.roussignol@univ-paris-est.fr

Michel Roussignol is a professor emeritus who serves on the Paris Est Marne la Vallée University. He works on the field of mathematics for reliability. He is the co-director thesis of William Lair.

Rachid Ziani, Project Manager,
Innovation & Research Department of SNCF
45 rue de Londres
75379 Paris France

e-mail : rachid.ziani@sncf.fr

Rachid Ziani is a project manager and works in the Innovation and Research Department of the SNCF. His research field is the statistic and reliability. He frames the thesis of William Lair.